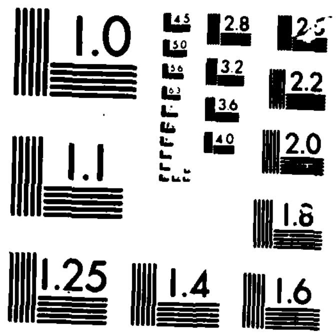
AD-A169 251 ASYMPTOTIC AND NUMERICAL METHODS FOR SINGULARLY PERTURBED DIFFERENTIAL EQ..(U) RENSSELAER POLYTECHNIC INST TROY NY DEPT OF MATHEMATICAL SCIE.. 1/1

UNCLASSIFIED J E FLAHERTY ET AL. 19 MAY 86 F/G 12/1 NL



MICROCOP' CHART

MASTER COPY - FOR REPRODUCTION PURPOSES

(2)

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

AD-A169 251

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER ARO 19512.17-MA	2. GOVT ACCESSION NO. N/A	3. RECIPIENT'S CATALOG NUMBER N/A
4. TITLE (and Subtitle) Asymptotic and Numerical Methods for Singularly Perturbed Differential Equations with Applications to Impact Problems		5. TYPE OF REPORT & PERIOD COVERED Final Report 10/1/82 - 1/31/86
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Joseph E. Flaherty Robert E. O'Malley		8. CONTRACT OR GRANT NUMBER(s) DAAG29-82-K-0197
9. PERFORMING ORGANIZATION NAME AND ADDRESS Rensselaer Polytechnic Institute Departments of Mathematical and Computer Science Troy, New York 12180-3590		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS U. S. Army Research Office Post Office Box 12211 Research Triangle Park, NC 27709		12. REPORT DATE 5/19/86
		13. NUMBER OF PAGES
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

DTIC ELECTE JUL 0 3 1986 D

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

NA

E

18. SUPPLEMENTARY NOTES

The view, opinions, and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

We developed and applied numerical methods for singularly perturbed two-point boundary value problems and time-dependent partial differential equations.

86 7 1 021

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

ARO 19512.17-MA

FINAL REPORT

U. S. Army Research Office Contract No. DAAG29-82-K-0197

Period: 1 October 1982 through 31 January 1986

Title of Research: Asymptotic and Numerical Methods for Singularly Perturbed Differential Equations with Applications to Impact Problems

Principal Investigators: Joseph E. Flaherty
Robert E. O'Malley, Jr.

Department of Mathematical Sciences
Rensselaer Polytechnic Institute
Troy, New York 12180

1. Research on Numerical Methods for Singularly Perturbed Differential Equations

During the period of this contract, we developed and applied numerical methods for singularly perturbed two-point boundary value problems and time-dependent partial differential equations. Our progress and publications in these areas are described below.

1.1. Two Point Boundary Value Problems

Our research during this period has largely centered on analyzing the asymptotic behavior of solutions to scalar problems of the form

$$\epsilon y'' + f(y)y' + g(y) = 0, \; 0 \leqslant X \leqslant c$$

with $y(0)=A$ and $y(c)=B$ prescribed. Such problems and their generalizations model a variety of nonlinear oscillators and other physical systems involving narrow boundary and interior layers where solutions change rapidly. A most fascinating example involves $f(y) = y(1-y^2)$ and $g(y) = -y$. The unique solution differs in twenty-two different regions of the A-B plane, having up to four different limiting solutions in each (depending on the interval length c). Away from regions of nonuniform convergence, the limiting solution $Y_0(x)$ will satisfy one of the five limits: $(1-Y_0^2)\dot{Y}_0 = 1, Y_0(0)=A$; $(1-Y_0^2)\dot{Y}_0 = 1, Y_0(0)=-1$; $(1-Y_0^2)\dot{Y}_0 = 1, Y_0(c)=B$; $(1-Y_0^2)\dot{Y}_0 = 1, Y_0(c)=1$; or $Y_0(x)=0$.

This behavior and the location of jump points can be explained by analyzing trajectories for the corresponding fast-slow system in the y-z phase plane, where z is the Lienard variable $z = \epsilon \dot{y} + F(y)$, $F(y) = \int^{y} f(s)\,ds$. More complicated behavior will be obtained for systems with more rest points and when the "characteristic curve" $z = F(y)$ has less symmetry and more extreme points. Our results may be typical of nonlinear problems which arise as reaction-diffusion systems or as other problems with slow/fast dynamics. They would be nearly impossible to comprehend and discover computationally, without some prior knowledge of possible analytical behavior. This work is being carried out with graduate student John Allen. It has benefited from graphical ODE routines produced by G. Odell. Further computational efforts will benefit from use of adaptive numerical methods, as have been developed by Flaherty, Brown and Lorenz, and others, and, ultimately, by devising domain decomposition techniques related to identification of asymptotic scales.

Vasil'eva and Butuzov, and Flaherty and O'Malley have independently developed analyses for singular-singular perturbation problems. These problems occur when the limiting equation has a family of solutions. They arise in

applications ranging from chemical kinetics to circuit theory. Recently, proposed simulation methods for rapidly oscillating controls made clear the need for a more comprehensive theory. This is the focus of thesis work by Gu Zhong-Mei and of a collaboration with Dr. Nikolai Nefedov, a visiting faculty member from Moscow State University. At this time, it appears that the Flaherty-O'Malley procedure (which provides a differential equation for the limiting solution) is most practical. However, more practical means of decoupling fast and slow parts of solutions and of examining appropriate stability manifolds for stretched systems are both very much needed. Results obtained will form a basis for regularization methods to solve differential- algebraic systems, as well as the original singular problems.

Other efforts by Professor O'Malley include continuing interactions with G. Soderlind of Stockholm's Royal Institute of Technology, R. Mattheij of the Catholic University of Nijmegen, and with colleagues J. Cole, J. Flaherty, H. Kurland, and M. Slemrod, among others. A fairly extensive set of class notes on singular perturbation methods was written last semester. It may soon form the basis of a research level monograph or graduate textbook.

1.2 Initial Boundary Value Problems for Partial Differential Equations

Professor Flaherty and several colleagues and graduate students have been conducting research on adaptive finite difference and finite element methods for partial differential equations.

Drew and Flaherty [2][1] created a model for shear band instabilities that involves the rapid shearing of a slab of a visco-elastic material with small viscosity. They used a moving mesh finite element code to solve their model and illustrated several possible mechanisms for shear band formations. In a somewhat related study, Slemrod and Flaherty [12] considered appropriate finite difference schemes for phase transitions involving van der Waals fluids.

Moving mesh methods were studied and the stability of several schemes that are based on equidistributing a local error indicator was characterized. Our findings, reported in [11], show that many popular mesh moving strategies can be unstable in certain situations.

Moving grid methods are effective at reducing dispersive errors in the vicinity of wave fronts; however, no method that uses a fixed number of computational cells can be used to compute a solution to a prescribed level of accuracy. For this reason, we have been studying and developing techniques that perform explicit error estimations

[1] See list of publications and manuscripts in Section 3

that are used to add and delete elements as the temporal integration progresses. In this spirit, Adjerid and Flaherty [13] developed a finite element method of lines where the mesh "lines" are adaptively moved to minimize artificial diffusion. but where elements are added and deleted during the integration. Piecewise linear finite elements are used to calculate the numerical solution and quadratic elements are used to estimate its error. Mesh motion is accomplished using one of the stable schemes studied in [11]. Ordinary differential equations are obtained for the finite element solution. the error estimate. and the mesh coordinates and these may be solved using existing software for stiff systems. We have applied this procedure to several problems. including some difficult combustion studies. and found some very encouraging results.

The elementary global refinement strategy that was used in [13] could be inefficient in many situations. Additionally. the mesh moving algorithm of [13] required users to specify a parameter that controlled relaxation times to an "ideal" mesh that equidistributes local error estimates. We have remedied both of these deficiencies and are now using an efficient local refinement strategy and an adaptive procedure for selecting the mesh relaxation parameter. The details of these algorithms and the results of several computational experiments have been incorporated into a manuscript which has been accepted for publication in *Comput. Meths. in Appl. Mech. Engr.* [16]. We have also extended our solution. error estimation. and local refinement procedures to two-dimensional parabolic problems. Details of the method. which uses dynamic tree structures to store data. and the results of several academic and practical examples were reported in Slimane Adjerid's Ph.D. dissertation [8].

The approach of Adjerid and Flaherty [13] couples mesh motion and error estimation to the solution. This produces a method with great numerical stability: however. there are many instances where this extra effort is not necessary. Thus. we are considering adaptive local refinement techniques that use rectangular and trapezoidal space-time elements (cf. Flaherty and Moore [3.6] and Bieterman et al. [14]). These methods discretize and solve a problem for one time step using a finite element-Galerkin procedure. At the end of the time step. the discretization error is estimated. finer subgrids of space-time elements are added to regions of high error. and the problem is recursively solved again on these regions. The process terminates and the integration continues to the next time step when the estimated error on each grid is less than a prescribed tolerance.

There is more to be done with our space-time local refinement codes. Flaherty and graduate students M. J. Coyle and P. K. Moore are concentrating on developing hierarchic error estimation techniques that are similar to those of Adjerid and Flaherty [13.16] for the method of lines. The essential difficulties are (i) the absence

of any superconvergence in time and (ii) the appropriate initial and boundary conditions to apply at coarse-fine mesh interfaces. In addition, Jackson and Flaherty [17] have extended the finite element basis to include discontinuous trial functions. They used these in conjunction with a moving mesh scheme of Harten and Hyman[2] to develop a method that is extremely well suited to the solution of hyperbolic systems with shocks. The method captures shocks and contact surfaces as true discontinuities without any artificial diffusion or oscillations.

We have developed an adaptive finite difference method for two-dimensional systems of conservation laws. Preliminary work on a technique that used MacCormack's method on a moving grid of quadrilateral elements was reported by Arney and Flaherty [5,15]. This mesh moving technique used a clustering algorithm of Berger[3] to locate and group regions of high error into rectangles that isolate spatially distinct phenomena. An algebraic mesh moving function was then used to move and align the mesh with the regions of high error. This procedure is very efficient and, unlike many other two-dimensional mesh moving techniques, it has no problem dependent parameters.

Major D. C. Arney, a professor at the U.S. Military Academy, completed his Ph.D. studies in 1985 under Flaherty's direction. He has combined the technique of [5,15] with a recursive local refinement strategy and used it to create an adaptive finite difference code for nonlinear two-dimensional hyperbolic systems [9]. The code has been applied to several difficult compressible flow problems that contain shocks, contact surfaces, and expansions. It appears to be able to utilize the best features of mesh moving and refinement.

2. Recent Interactions

Professors Flaherty and O'Malley and graduate students lectured and/or visited the following conferences and organizations during 1985.

J. E. Flaherty visited Los Alamos National Laboratory, Los Alamos, 20-21 June, 1985. He held technical discussions with Drs. J. M. Hyman, T. Manteufel, J. Dendy, B. Swartz, and B. Wendroff and lectured on "Moving Finite Difference and Finite Element Methods with Local Refinement for Time Dependent Partial Differential Equations."

[2] A. Harten and J. M. Hyman, "Self Adjusting Grid Methods for One-Dimensional Hyperbolic Conservation Laws," **J. Comp. Phys.**, Vol. 50, 269-325, 1983.

[3] M. J. Berger, "Adaptive Mesh Refinement for Hyperbolic Partial Differential Equations," Report No. STAN-CS-82-924, Department of Computer Science, Stanford University, 1982.

J. E. Flaherty attended the ASME/ASCE Applied Mechanics Conference. Albuquerque, 24-26 June. 1985. He presented an invited lecture on "Local Refinement and Moving Finite Element Methods for Parabolic Partial Differential Equations" as part of the Symposium on Recent Advances in Computational Mechanics. He also had technical discussions with Profs. R. Ewing of Wyoming University. J. T. Oden of the University of Texas, and R. D. Russell of Simon Frasier University.

R. E. O'Malley attended the AMS Symposium on Combustion Theory at Cornell University for a week in July. Especially valuable correspondence concerning singular models for chemical kinetics resulted with Professor S.-H. Lam of Princeton University.

R. E. O'Malley visited Los Alamos National Laboratory and its Center for Nonlinear Studies for a week in August. He had technical discussions about singular perturbations with B. Nicolaenko. P. Fife, D. Brown. D. Cohen. J. M. Hyman. D. McLaughlin, and J. D. Murray. among others.

J. E. Flaherty co-organized (with D. A. Drew) the Conference on Mathematics Applied to Fluid Mechanics and Stability which was dedicated in memory of Professor R. C. Di Prima and held at RPI, 9-11 September, 1985.

J. E. Flaherty visitied the IBM T. J. Watson Research Center. Yorktown Heights. 9 October. 1985 and lectured on "Adaptive Finite Difference and Finite Element Methods for Time Dependent Partial Differential Equations." He also had technical discussions with Drs. W. Liniger and F. Odeh.

J. E. Flaherty visited the Courant Institute of Mathematical Sciences. New York. 18 October. 1985. He had technical discussions with Profs. M. Berger and P. Collela and lectured on "Adaptive Finite Difference and Finite Element Methods for Time Dependent Partial Differential Equations."

S. Adjerid and J. E. Flaherty attended the SIAM Fall meeting. Arizona State University. Tempe. 28-30 October. 1985. They had technical discussions with several individuals. including Dr. R. C. Y. Chin of Lawrence Livermore Laboratories. Prof. R. Bank of the University of California at San Diego. and Prof. R. D. Russell of Simon Frasier University.

Professor O'Malley also attended the SIAM Fall meeting in Tempe. This provided an opportunity for discussions about numerical methods for two-point stiff problems with G. Dahlquist. R. Russell. C. Ringhofer. and C. Schmeisser. among others. In October. he gave a seminar on the subject at the University of Montreal and a series of three lectures at the University of New Hampshire.

S. Adjerid, J. E. Flaherty, and P. K. Moore attended the SIAM Conference on Parallel Processing for Scientific Computing, Norfolk, 18-21 November, 1985. They had technical discussions with several individuals including Maj. D. C. Arney of the United States Military Academy, Drs. R. C. Y. Chin and L. Petzold of Lawrence Livermore Laboratories, Dr. S. F. Davis of the Naval Surface Weapons Center, and Prof. J. Van Rosendale of the University of Utah.

S. Adjerid, J. E. Flaherty, and P. K. Moore attended a day-long workship on adaptive methods for partial differential equations at the University of Maryland on 24 January 1986. Each of them presented a lecture on adaptive methods. Also present at the workshop were I. Babuska, J. Osborne, and E. Rank of the University of Maryland; P. Baehman and M. Shephard of RPI; J. Thomas of AFOSR; S. F. Davis and W. Szymczak of the Naval Surface Weapons Center; and other students and faculty.

Professor O'Malley is chairman of the 1986 SIAM National Meeting. Its program will feature many sessions relevant to this contract. He is also organizing a SIAM Conference on Numerical Methods for Singularly Perturbed Problems.

Effective January 1, 1986, Professor O'Malley is SIAM's Vice President for Publications. He has also joined the editorial board of the IMA Journal on Applied Mathematics.

Drs. O'Malley and Nefedov visited California in February to discuss singular perturbation problems with Professors Cohen, Lorenz, Lagerstrom, and Kreiss at Caltech; Professors Harris, Sibuya, and Sacker at the University of Southern California; and Drs. Howes and Chin at the Lawrence Livermore Laboratories.

3. List of Publications and Manuscripts

Publications:

1. R. E. O'Malley, Jr. and J. E. Flaherty, "On the Numerical Solution of Singularly-Perturbed Vector Boundary Value Problems," in *Trans. Tenth IMACS World Cong. on Syst. Simul. and Sci. Comput.*, R. Stepelman, M. Carver, R. Peskin, W. F. Ames, R. Vichnevetsky (Eds.), Vol. 1, North-Holland, New York, pp. 105-112, 1983.

2. D. A. Drew and J. E. Flaherty, "Adaptive Finite Element Methods and the Numerical Solution of Shear Band Problems," in *Phase Transitions and Material Instabilities in Solids*, M. Gurtin, Ed., Academic Press, New York, pp. 37-60, 1984.

3. J. E. Flaherty and P. K. Moore, "An Adaptive Local Refinement Finite Element Method for Parabolic Partial Differential Equations," in *Proc. Int. Conf. on Accuracy Estimates and Adaptive Refinements in Finite Element Computations,* Int. Assn. Compu. Mech., Vol. 2, Lisbon, pp. 139-152, 1984.

4. J. E. Flaherty, M. L. Frankel, V. Roytburd, G. I. Sivaskinsky, "Numerical Study of Quenching of Inward Propagating Spherical Flames," in *Combust. Sci. Tech.*, Vol. 43, pp. 245-257, 1985.

5. D. C. Arney and J. E. Flaherty, "A Mesh Moving Technique for Time Dependent Partial Differential Equations in Two-Space Dimensions," in *Trans. Second Army Conf. on Appl. Maths. and Comput.*, U. S. Army Research Office Report No. ARO 85-1, U. S. Army Research Office, Research Triangle Park, pp. 611-634, 1985.

6. J. E. Flaherty and P. K. Moore, "A Local Refinement Finite Element Method for Time Dependent Partial Differential Equations," in *Trans. Second Army Conf. on Appl. Maths. and Comput.*, U. S. Army Research Office Report No. ARO 85-1, U. S. Army Research Office, Research Triangle Park, pp. 585-596, 1985.

7. R. E. O'Malley, Jr., "On the Simultaneous Use of Asymptotic and Numerical Methods to Solve Nonlinear Two-point Problems with Boundary and Interior Layers," in *Numerical Boundary Value ODEs*, U. M. Ascher and R. D. Russell (Eds.), Birkhauser, Boston, pp. 149-172, 1985.

8. S. Adjerid, "Adaptive Finite Element Methods for Time Dependent Partial Differential Equations," Ph.D. Dissertation, Department of Mathematical Sciences, Rensselaer Polytechnic Institute, Troy, 1985.

9. D. C. Arney, "An Adaptive Mesh Algorithm for Solving Systems of Time Dependent Partial Differential Equations," Ph.D. Dissertation, Department of Mathematical Sciences, Rensselaer Polytechnic Institute, Troy, 1985.

10. R. E. O'Malley, Jr., and R. M. M. Mattheij, "Decoupling and Boundary Value Problems for Two-Time-Scale Systems," *Stiff Computation*, R. C. Aiken (Ed.), Oxford University Press, pp. 104-111, 1985.

11. J. M. Coyle, J. E. Flaherty and R. Ludwig, "On the Stability of Mesh Equidistribution Strategies for Time Dependent Partial Differential Equations," *J. Comp. Phys.*, Vol. 62, pp. 26-39, 1986.

In Press:

12. M. Slemrod and J. E. Flaherty, "Numerical Integration of a Riemann Problem for a van der Waals Fluid," *Res. Mechanica*, accepted for publication, 1984.

13. S. Adjerid and J. E. Flaherty, "A Moving Finite Element Method with Error Estimation and Refinement for One-Dimensional Time Dependent Partial Differential Equations," *SIAM J. Numer. Anal.*, accepted for publication, 1985.

14. M. Bieterman, J. E. Flaherty, and P. K. Moore, "Adaptive Refinement Methods for Nonlinear Parabolic Partial Differential Equations," I. Babuska, O. C. Zienkiewicz, E. Arantes e Oliveira, J. R. Gago, and K. Morgan, (Eds.), *Accuracy Estimates and Adaptivity for Finite Elements*, John Wiley, London, accepted for publication, 1984.

15. D. C. Arney and J. E. Flaherty, "A Two-Dimensional Mesh Moving Technique for Time Dependent Partial Differential Equations," *J. Comp. Phys.*, accepted for publication, 1986.

16. S. Adjerid and J. E. Flaherty, "A Moving Mesh Finite Element Method with Local Refinement for Parabolic Partial Differential Equations," *Computer Methods in Applied Mechanics and Engineering*, accepted for publication, 1985.

17. J. E. Flaherty and T. L. Jackson, "A Discontinuous Finite Element Method for Hyperbolic Systems of Conservation Laws," submitted to *SIAM J. Sci. Stat. Comput.*, September 1985.

18. "Singular Perturbation in Systems and Control," *IEEE Press*, 1986, (P. V. Kokotovic and H. K. Khalil, (Eds.), will reprint articles by Kokotovic, Sannuti, and O'Malley (Automatica 1976), O'Malley (SIAM J. Control, 1972), O'Malley (J. Math. Anal. Appl., 1974), and O'Malley and Jameson (IEEE Trans. Automatic Control, 1977).

19. R. E. O'Malley, Jr., "On Nonlinear Singularly Perturbed Initial Value Problems," submitted to *SIAM Review*, July 1985.

ON THE NUMERICAL SOLUTION OF SINGULARLY-PERTURBED VECTOR BOUNDARY VALUE PROBLEMS

Robert E. O'Malley, Jr.
and
Joseph E. Flaherty
Department of Mathematical Sciences
Rensselaer Polytechnic Institute
Troy, New York 12180 USA

ABSTRACT

Numerical procedures are developed for constructing asymptotic solutions of certain nonlinear singularly-perturbed vector two-point boundary value problems having boundary layers at one or both end points. The asymptotic approximations are generated numerically and can either be used as is or to furnish a two-point boundary value code (e.g., COLSYS) with an initial approximation and a nonuniform computational mesh. The procedures are applied to a model problem that indicates the possibility of multiple solutions and problems involving the deformation of a thin nonlinear elastic beam resting on a nonlinear elastic foundation.

In **Trans. Tenth IMACS World Cong. on Syst. Simul. and Sci. Comput.**, R. Stepleman, et al (Eds.), Vol. 1, North-Holland, New York, pp. 105-112, 1983

ADAPTIVE FINITE ELEMENT METHODS AND THE NUMERICAL SOLUTION OF SHEAR BAND PROBLEMS

Donald A. Drew and Joseph E. Flaherty
Department of Mathematical Sciences
Rensselaer Polytechnic Institute
Troy, New York 12180

ABSTRACT

Shear bands are localized regions of very high shear strain which arise as a result of high rates of loading. They occur in metal forming and cutting processes and in impact and penetration problems. In this paper, we describe a model for the formation of shear bands in simple shear that involves the description of irreversible mechanical shear and the resulting heat release. The location of a shear band is unknown in advance, and the evolution results in large gradients of displacement, velocity, and temperature. Shear band formation, therefore, offers an interesting and physically important application of a code able to resolve small-scale transient structures. In this paper, we use an adaptive finite element code to solve several problems involving shear band formation. The code automatically locates regions with large gradients and adaptively concentrates finite elements there in order to minimize approximately the development of shear bands under many circumstances and indicate some possible mechanisms for their formation.

In **Phase Transitions and Material Instabilities in Solids**, M. Gurtin (Ed.), Academic Press, New York, pp. 37-60, 1984

AN ADAPTIVE LOCAL REFINEMENT FINITE ELEMENT METHOD FOR PARABOLIC PARTIAL DIFFERENTIAL EQUATIONS

Joseph E. Flaherty
Department of Computer Science
Rensselaer Polytechnic Institute
Troy, New York 12180
and
Peter K. Moore
Department of Mathematical Sciences
Rensselaer Polytechnic Institute
Troy, New York 12180

ABSTRACT

We discuss an adaptive finite element method for solving initial-boundary value problems for vector systems of partial differential equations in one space dimension and time. The method uses piecewise bilinear rectangular space-time finite elements. For each time step, the grid is automatically refined in regions where the local discretization error is estimated as being larger than a prescribed tolerance. We discuss several aspects of our algorithm, including the tree structure that is used to represent the finite element solution and grids, an error estimation technique, and initial and boundary conditions at coarse-fine mesh interfaces. We also present the results of several computational examples and experiments.

In **Proc. Int. Conf. on Accuracy Estimates and Adaptive Refinements in Finite Element Computations.** Int. Assn. Compu. Mech., Vol. 2, Lisbon, pp. 139-152, 1984

NUMERICAL STUDY OF QUENCHING OF INWARD PROPAGATING SPHERICAL FLAMES

J. E. Flaherty
Department of Computer Science
Rensselaer Polytechnic Institute
Troy, New York 12180
and
M. L. Frankel, V. Roytburd
Department of Mathematical Sciences
Rensselaer Polytechnic Institute
Troy, New York 12180
and
G. I. Sivashinsky
Department of Mathematical Sciences
Tel-Aviv University
Tel-Aviv 69978, Israel

ABSTRACT

The phenomenon of quenching of cylindrical convergen flames is considered in the framework of constant density approximation. The parameters dependence of the effect is studied numerically using an adaptive finite element method. The numerical results are in a good agreement with predictions of theoretical analysis from (Frankel and Sivashinsky, 1984).

In **Combust. Sci. Tech.**, Vol. 43, pp. 245-257, 1985

A MESH MOVING TECHNIQUE FOR TIME DEPENDENT PARTIAL DIFFERENTIAL EQUATIONS IN TWO SPACE DIMENSIONS

David C. Arney
Department of Mathematics
United States Military Academy
West Point, NY 10996
and
Department of Mathematical Sciences
Rensselaer Polytechnic Institute
Troy, NY 12180
and
Joseph E. Flaherty
Applied Mathematics and Mechanics Section
Benet Weapons Laboratory
Watervliet, NY 12189
and
Department of Computer Science
Rensselaer Polytechnic Institute
Troy, NY 12180

ABSTRACT

We discuss an adaptive mesh moving technique that is used with a finite difference or finite element scheme to solve initial-boundary value problems for vector systems of partial differential equations in two space dimensions and time. The mesh moving technique is based on an algebraic node movement function determined from the *propagation of significant error regions*. The algorithm is designed to be flexible, so that it can be used with many existing finite difference or finite element methods. To test the algorithm, we implemented the mesh mover in a system code along with an initial mesh generator and a MacCormack finite volume integrator to solve hyperbolic vector systems. Results are presented for several computational examples. The moving mesh reduces dispersion errors near shocks and wave fronts and thereby can reduce the grid requirements necessary to compute accurate solutions while increasing computational efficiency.

In **Trans. Second Army Conf. on Appl. Maths. and Comput.**, ARO Report 85-1, U.S. Army Research Office, Research Triangle Park, pp. 611-634, 1985

A LOCAL REFINEMENT FINITE ELEMENT METHOD FOR TIME DEPENDENT PARTIAL DIFFERENTIAL EQUATIONS

Joseph E. Flaherty
Department of Computer Science
Rensselaer Polytechnic Institute
Troy, N.Y. 12180
and
U. S. Army Armament Research and Development Command
Large Calibre Weapon Systems Laboratory
Benet Weapons Laboratory
Watervliet, NY 12189
and
Peter K. Moore
Department of Mathematical Sciences
Rensselaer Polytechnic Institute
Troy, NY 12180

ABSTRACT

We discuss an adaptive finite element method for solving initial-boundary value problems for vector systems of partial differential equations in one space dimension and time. The method uses piecewise bilinear rectangular space-time finite elements. For each time step, the grid is automatically refined in regions where the local discretization error is estimated as being larger than a prescribed tolerance. We discuss several aspects of our algorithm, including the tree structure that is used to represent the finite element solution and grids, an error estimation technique, and initial and boundary conditions at coarse-fine mesh interfaces. We also present the results of several computational examples and experiments.

In **Trans. Second Army Conf. on Appl. Maths. and Comput.**, ARO Report 85-1, U.S. Army Research Office, Research Triangle Park, pp. 585-596, 1985

ON THE SIMULTANEOUS USE OF ASYMPTOTIC AND NUMERICAL METHODS TO SOLVE NONLINEAR TWO-POINT PROBLEMS WITH BOUNDARY AND INTERIOR LAYERS

R. E. O'Malley, Jr.
Department of Mathematical Sciences
Rensselaer Polytechnic Institute
Troy, New York 12180

ABSTRACT

The purpose of this paper is to provide a survey concerning boundary value problems for certain systems of nonlinear singularly perturbed ordinary differential equations. The aim is to emphasize important and difficult open problems needing more study, in terms of both mathematical and numerical analysis and computational experiments. Some currently tractable problems are discussed in detail.

In **Numerical Boundary Value ODEs**, U. Ascher and R. Russell (Eds.), Birkhauser, Boston, pp. 149-172, 1985.

ADAPTIVE FINITE ELEMENT METHODS FOR TIME DEPENDENT PARTIAL DIFFERENTIAL EQUATIONS

Slimane Adjerid
Department of Mathematical Sciences
Rensselaer Polytechnic Institute
Troy, New York 12180

ABSTRACT

We discuss adaptive finite element methods for solving initial-boundary problems for vector systems of partial differential equations in one and two space dimensions.

One dimensional systems are discretized using piecewise linear finite element approximations in space and a backward difference code for stiff ordinary differential systems in time. A spatial error estimate is calculated using piecewise quadratic approximations that use the superconvergence property of parabolic systems to gain computational efficiency. This error estimate is used to move and refine the finite element mesh in order to equidistribute a measure of the total spatial error and to satisfy a prescribed error tolerance. Ordinary differential equations for the spatial error estimate and the mesh motion are integrated in time using the same backward difference software that is used to determine the numerical solution of the partial differential system. The one-dimensional algorithm combines mesh moving with local refinement in a relatively efficient manner and attempts to eliminate problem-dependent numerical parameters. A variety of examples that motivate our mesh moving strategy and illustrate the performance of our algorithms are presented.

Two-dimensional systems are discretized using piecewise bilinear finite element approximations in space and the same backward difference software that is used for one-dimensional systems in time. A spatial error estimate is calculated using piecewise cubic approximations that use the superconvergence property of parabolic systems. This error estimate is used to locally refine a stationary finite element mesh in order to satisfy a prescribed spatial error tolerance.

Several linear and nonlinear examples are presented to illustrate the effectiveness of our error estimation technique and the performance of our adaptive algorithms.

Ph.D. Dissertation, Department of Mathematical Sciences, Rensselaer Polytechnic Institute, Troy, 1985

AN ADAPTIVE MESH ALGORITHM FOR SOLVING SYSTEMS OF TIME DEPENDENT PARTIAL DIFFERENTIAL EQUATIONS

David C. Arney
Department of Mathematics
United States Military Academy
West Point, NY 10996
and
Department of Mathematical Sciences
Rensselaer Polytechnic Institute
Troy, New York 12180

ABSTRACT

We discuss an adaptive mesh algorithm that can be used with a finite difference or finite element scheme to solve initial-boundary value problems for vector systems of time dependent partial differential equations in two space dimensions. Our algorithm combines the adaptive techniques of mesh moving, static rezoning, and local mesh refinement. The nodes of a coarse mesh of quadrilateral cells are moved by a simple algebraic node movement function, determined from the geometry and propagation of regions having statically significant discretization error or mesh movement indicators. The local mesh refinement method recursively divides cells of the moving coarse mesh within clustered regions that contain nodes with large error until a user prescribed error tolerance is satisfied. These finer grids are properly nested within the moving coarse mesh to provide for simpler data structures and interface conditions between the fine and coarse meshes.

Our procedure is designed to be flexible, so that it can be used with many existing finite difference and finite element schemes and with different error estimation procedures. To test our adaptive mesh algorithm, we implemented it in a system code with an initial mesh generator, a MacCormack finite difference scheme for hyperbolic vector systems of conservation laws, and a Richardson extrapolation based error estimation. Results are presented for several computational examples.

The moving mesh technique reduces dispersive errors near shocks and wave fronts. Therefore, it reduces the grid requirements necessary to compute accurate solutions and thus increases computational efficiency. The local mesh refinement provides smaller mesh spacings and time steps in regions where the problem is difficult to solve, thus providing increased accuracy and enabling error tolerances to be achieved.

Ph.D. Dissertation, Department of Mathematical Sciences, Rensselaer Polyechnic Institute, Troy, 1985

DECOUPLING AND BOUNDARY VALUE PROBLEMS FOR TWO-TIME-SCALE SYSTEMS

R. E. O'Malley, Jr.
Department of Mathematical Sciences
Rensselaer Polytechnic Institute
Troy, New York 12180
and
R. M. M. Mattheij
Catholic University
Nijmegen, The Netherlands

ABSTRACT

This paper presents an analytical technique for decoupling slow and fast dynamics for linear systems of ordinary differential equations. The method, based on Riccati transformations, is of substantial computational value for solving stiff two-point boundary value problems.

In **Stiff Computation**, R. C. Aiken (Ed.), Oxford University Press, pp. 104-111, 1985.

ON THE STABILITY OF MESH EQUIDISTRIBUTION STRATEGIES FOR TIME DEPENDENT PARTIAL DIFFERENTIAL EQUATIONS

J. Michael Coyle, Joseph E. Flaherty and Raymond Ludwig
Department of Mathematical Sciences
Rensselaer Polytechnic Institute
Troy, New York 12180 USA

ABSTRACT

We study the stability of several mesh equidistribution schemes for time dependent partial differential equations in one space dimension. The schemes move a finite difference or finite element mesh so that a given quantity is uniform over the domain. We consider mesh moving methods that are based on solving a system of ordinary differential equations for the mesh velocities and show that many of these methods are unstable with respect to an equidistributing mesh when the partial differential system is dissipative. Using linear perturbation techniques, we are able to develop simple criteria for determining the stability of a particular method and show how to construct stable differential systems for the mesh velocities. Several examples illustrating stable and unstable mesh motions are presented.

In **J. Comp. Phys.**, Vol. 62, pp. 26-39, 1986

NUMERICAL INTEGRATION OF A RIEMANN PROBLEM FOR A VAN DER WAALS FLUID

M. Slemrod
and
J. E. Flaherty
Department of Mathematical Sciences
Rensselaer Polytechnic Institute
Troy, New York 12180

ABSTRACT

In two recent papers, Slemrod has suggested that the well known Lax-Friedrichs finite difference method may provide a natural method for the numerical integration of initial value problems with an anomalous equation of state, e.g., a van der Waals fluid. In this note we review these ideas and present the results of a numerical experiment which attempts to simulate the dynamics of a van der Waals like fluid.

Submitted to **Res Mechanica**, April 1984

A MOVING FINITE ELEMENT METHOD WITH ERROR ESTIMATION AND REFINEMENT FOR ONE-DIMENSIONAL TIME DEPENDENT PARTIAL DIFFERENTIAL EQUATIONS

Slimane Adjerid
Department of Mathematical Sciences
Rensselaer Polytechnic Institute
Troy, New York 12180
and
Joseph E. Flaherty
Department of Computer Science
Rensselaer Polytechnic Institute
Troy, New York 12180

Dedicated in Memory of Richard C. DiPrima

ABSTRACT

We discuss a moving finite element method for solving vector systems of time dependent partial differential equations in one space dimension. The mesh is moved so as to equidistribute the spatial component of the discretization error in H^1. We present a method of estimating this error by using p-hierarchic finite elements. The error estimate is also used in an adaptive mesh refinement procedure to give an algorithm that combines mesh movement and refinement. We discretize the partial differential equations in space using a Galerkin procedure with piecewise linear elements to approximate the solution and quadratic elements to estimate the error. A system of ordinary differential equations for mesh velocities are used to control element motions. We use existing software for stiff ordinary differential equations for the temporal integration of the solution, the error estimate, and the mesh motion. Computational results using a code based on our method are presented for several examples.

To appear in **SIAM J. Numer. Anal.**, 1985

ADAPTIVE REFINEMENT METHODS FOR NONLINEAR PARABOLIC PARTIAL DIFFERENTIAL EQUATIONS

Michael Bieterman
Boeing Computer Services
Engineering Technology Applications
Tukwila, Washington 98118 USA
and
Joseph E. Flaherty
Department of Computer Science
Rensselaer Polytechnic Institute
Troy, New York 12180 USA
and
Peter K. Moore
Department of Mathematical Sciences
Rensselaer Polytechnic Institute
Troy, New York 12180 USA

ABSTRACT

We consider two adaptive finite element techniques for parabolic partial differential equations (PDEs) that are based on using error estimates to control mesh refinement. One technique is a method of lines (MOL) approach that uses a Galerkin method to discretize the PDEs in space and implicit multi-step integration in time. Spatial elements are added and deleted in regions of high and low error and are all advanced with the same sequence of varying time steps. The second technique is a local refinement method (LRM) that uses Galerkin approximations in both space and time. Fine grids of space-time elements are added to coarser grids and the problem is recursively solved in regions of high error.

To appear in **Accuracy Estimates and Adaptivity for Finite Elements**, I. Babuska, O. C. Zienkiewicz, E. Arantes e Oliveira, J. R. Gago and K. Morgan (Eds.), 1984

A TWO-DIMENSIONAL MESH MOVING TECHNIQUE FOR TIME DEPENDENT PARTIAL DIFFERENTIAL EQUATIONS

David C. Arney
Department of Mathematics
United States Military Academy
West Point, NY 10996
and
Department of Mathematical Sciences
Rensselaer Polytechnic Institute
Troy, New York 12180
and
Joseph E. Flaherty
Department of Computer Science
Rensselaer Polytechnic Institute
Troy, New York 12180

ABSTRACT

We discuss an adaptive mesh moving technique that can be used with a finite element difference or finite element scheme to solve initial-boundary value problems for vector systems of partial differential equations in two space dimensions and time. The mesh moving technique is based on an algebraic node movement function determined from the geometry and propagation of regions having significant discretization error indicators. Our procedure is designed to be flexible, so that it can be used with many existing finite difference and finite element methods. To test the mesh moving algorithm, we implemented it in a system code with an initial mesh generator and a MacCormack finite difference scheme on quadrilateral cells for hyperbolic vector systems of conservation laws. Results are presented for several computational examples. The moving mesh scheme reduces dispersive errors near shocks and wave fronts and thereby reduces the grid requirements necessary to compute accurate solutions while increasing computational efficiency.

To appear in **J. Comp. Phys.**, 1986

A MOVING MESH FINITE ELEMENT METHOD WITH LOCAL REFINEMENT FOR PARABOLIC PARTIAL DIFFERENTIAL EQUATIONS

Slimane Adjerid
Department of Mathematical Science
Rensselaer Polytechnic Institute
Troy, New York 12180
and
Joseph E. Flaherty
Department of Computer Science
Rensselaer Polytechnic Institute
Troy, New York 12180

ABSTRACT

We discuss a moving mesh finite element method for solving initial-boundary value problems for vector systems of partial differential equations in one space dimension and time. The system is discretized using piecewise linear finite element approximations in space and a backward difference code for stiff ordinary differential systems in time. A spatial error estimation is calculated using piecewise quadratic approximations that use the superconvergence properties of parabolic systems to gain computational efficiency. The spatial error estimate is used to move and locally refine the finite element mesh in order to equidistribute a measure of the total spatial error and to satisfy a prescribed error tolerance. Ordinary differential equations for the spatial error estimate and the mesh motion are integrated in time using the same backward difference software that is used to determine the numerical solution of the partial differential system.

We present several details of an algorithm that may be used to develop a general purpose finite element code for one-dimensional parabolic partial differential systems. The algorithm combines mesh motion and local refinement in a relatively efficient manner and attempts to eliminate problem-dependent numerical parameters. A variety of examples that motivate our mesh moving strategy and illustrate the performance of our algorithm are presented.

To appear in **Computer Methods in Applied Mechanics and Engineering**, 1985.

A DISCONTINUOUS FINITE ELEMENT METHOD FOR HYPERBOLIC SYSTEMS OF CONSERVATION LAWS

T. L. Jackson
Department of Mathematical Sciences
Rensselaer Polytechnic Institute
Troy, New York 12180
and
J. E. Flaherty
Department of Computer Science
Rensselaer Polytechnic Institute
Troy, New York 12180

ABSTRACT

We develop an adaptive finite element method for solving nonlinear hyperbolic systems of conservation laws in one space dimension and time. The method uses discontinuous piecewise linear trial functions and continuous piecewise cubic test functions on a moving mesh of triangular space-time elements. The mesh is moved by a technique that uses a weighted average of the local characteristic speeds to select nodal velocities. We show that discontinuous finite element approximations on a moving mesh have the potential of accurately resolving physical discontinuities in the solution. Several computational examples are presented to illustrate the performance of the method.

Submitted to **SIAM J. Sci. Stat. Comput.**, September 1985

ON NONLINEAR SINGULARLY PERTURBED INITIAL VALUE PROBLEMS*

R. E. O'Malley, Jr.
Department of Mathematical Sciences
Rensselaer Polytechnic Institute
Troy, New York 12180

ABSTRACT

The study of singularly perturbed initial value problems for nonlinear systems of ordinary differential equations parallels the analysis underlying the development of numerical algorithms for obtaining solutions of such systems of stiff differential equations. This paper seeks to emphasize the advantages of combining these two substantial research efforts. It develops insight and intuition based on a sequence of solvable model problems, and it relates a variety of literature scattered throughout publications concerned with asymptotic and numerical analyses, stability and control theory, and specific topics in applied mathematical modeling.

*This material was presented as an invited talk at the Joint U.S.-Scandinavian Symposium on Scientific Computing and Mathematical Modeling in Stockholm, January 1985. The work was supported in part by the National Science Foundation under Grant Number DMS-8301665 and by the U. S. Army Research Office under Grant Number DAAG29-82-K-0197.

END

DTIC

7-86